CS 133 - Introduction to Computational and Data Science

Instructor: Renzhi Cao
Computer Science Department
Pacific Lutheran University
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Probability

- Quiz #3.
- Check Sakai for exercises.
- In the previous class, we learned Statistics
- Today we are going to learn Probability

Probability

Quantifying the uncertainty associated with events chosen from a universe of events.

Universe: All possible outcomes

Event: A subset of those outcomes

Used to construct and evaluate models

"The laws of probability, so true in general, so fallacious in particular"

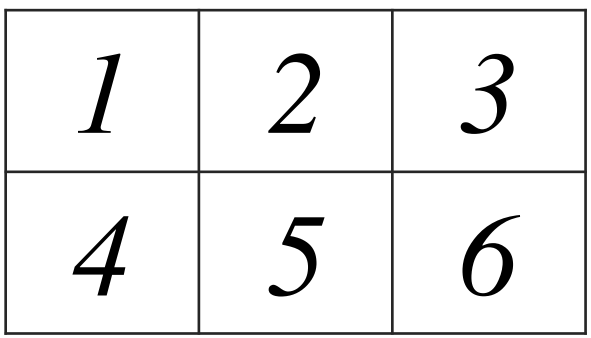
Edward Gibbon

Probability space

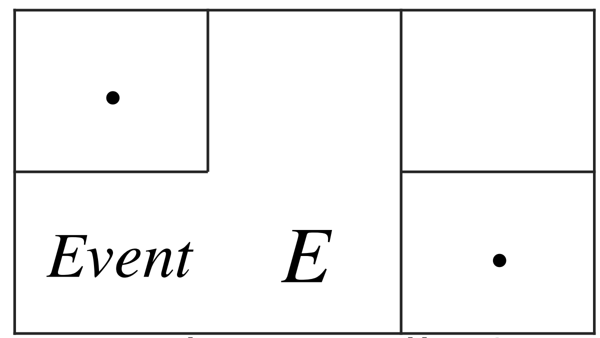
Finite set of points, whereby each of them represents a possible outcome of a specific experiment

- Each point (outcome) has a probability associated with it
- Probabilities are always positive!!!
- The sum of all probabilities is always 1
- Assume an equal probability distribution if not otherwise stated
  - e.g. 1/6 for a specific number of a die throw (unless die is not fair)

Probability space example



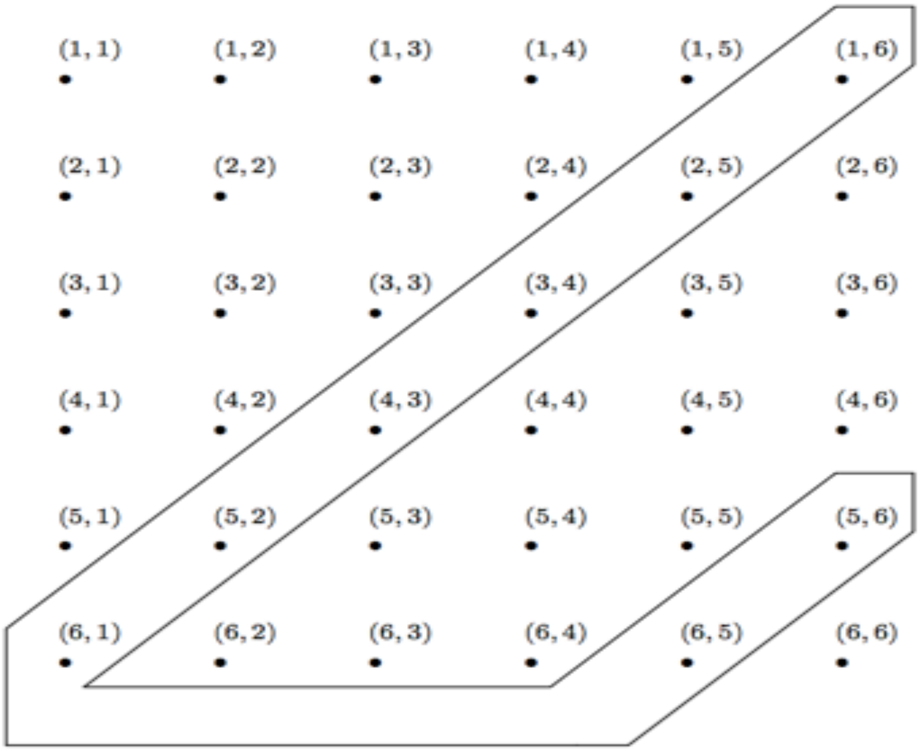
Probability space



Event E that you roll a 2, 4, or 5

Imagine you throw a dart randomly at the box. You will hit the area of E 50% of the time. P(E) = 0.5

Example: Craps



Throw 2 dice and calculate the probability of obtaining a total of 7 or 11.

36 possible outcomes

The event contains 8 points.

$p = 8/36 \approx 22\%$

Dependence and Independence

The probability of an event E: P(E)

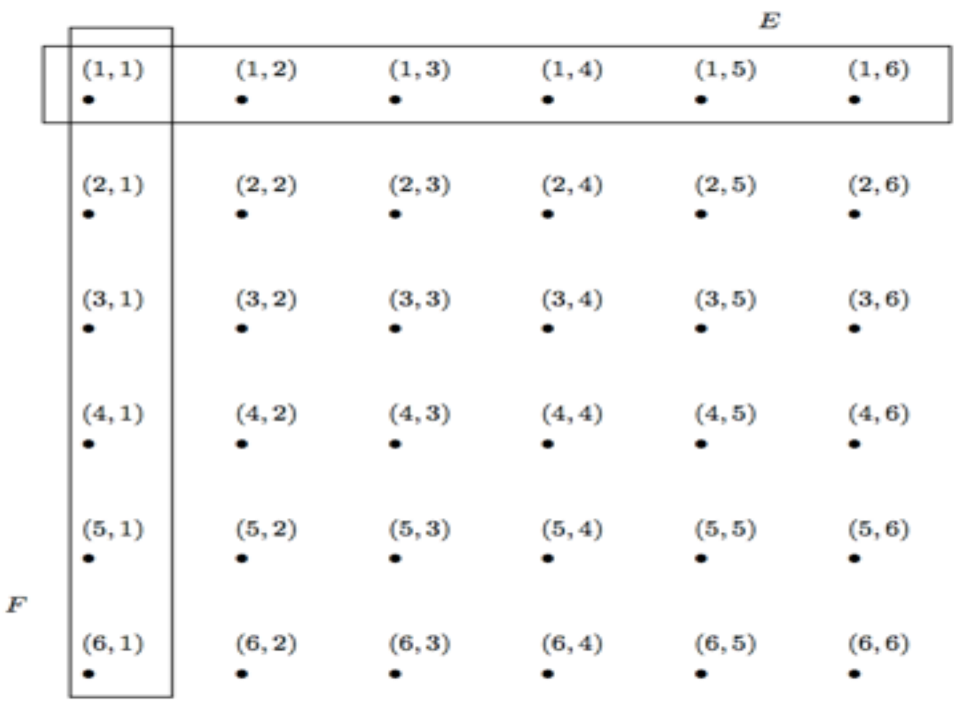
What about two events?

Events P and E are dependent if knowing something about whether E happens gives information about whether F happens.

Independent: The opposite

Tossing a coin two times: Dependent or independent?

Example 1: Conditional Probability



Toss of 2 dice

Probability space has 36 elements with equal probability 1/36

E: First comes out 1 (E_1)
F: Second comes out 1 (E_2)

P(E) = ? = 6/36 = 1/6
P(F) = ? = 6/36 = 1/6
P(F|E) = ? = 1/6

The experiments are **independent**, since **P(F) = P(F|E)**.
It does not matter if E occurred or not; the probability of F stays the same.

Independent Events

Independent Events: P(E,F) = P(E)P(F)

Example: Probability of getting two tails when flipping a coin two times. (Event E: first time gets tail. Event F: second time gets tail).

How to calculate P(E) and P(F)?

What is P(E,F)?

P(E,F) = ½ * ½ = ¼

Probability of getting a tail and a head:

Example 2: Conditional Probability [1]

Deal of 2 cards from a 52 card deck

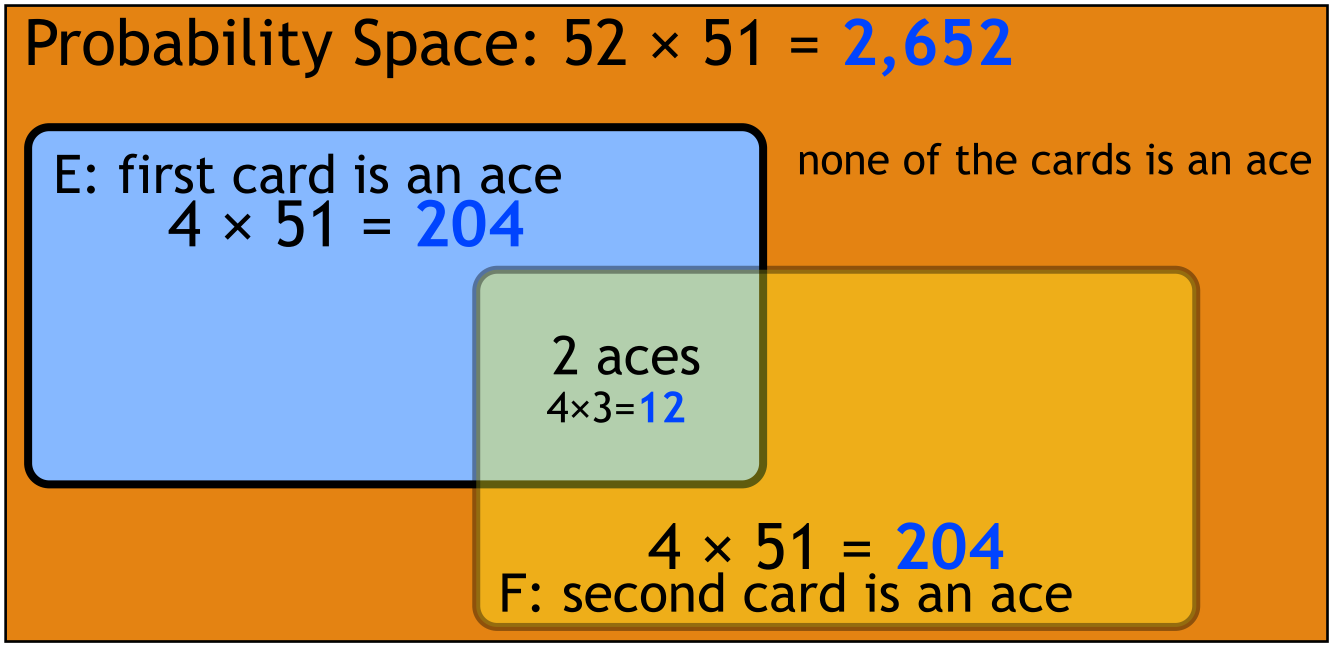
Number of points in experiment (probability space):
$\Pi(52,2) = 52 \times 51 =$ **2,652**

E: First card is an ace: $4 \times 51 =$ **204**
(4 choices for ace, 51 choices for second card)
$P(E) = 204/2{,}652 = 1/13$

F: Second card is an ace: $4 \times 51 =$ **204**
(4 choices for ace, 51 choices for first card)
$P(F) = 204/2{,}652 = 1/13$

$P(F|E) = 12/204 = 1/17$ $(= 3/51)$
since there are $4 \times 3 =$ **12** combinations for aces.

Example 2: Conditional Probability [1]



P(E) = 204/2,652

P(F) = 204/2,652

P(F|E) = 12/204

The experiments are **not independent**, since **P(F) ≠ P(F|E).**
It **does** matter if E occurred or not; the probability of F changes.

Dependent Events

Dependent Events: P(E|F) = P(E,F)/P(F), in which P(E|F)!=P(E)

Examples: There are 5 marbles in a bag. 3 green and 2 red.

P(1^{st} green) = ? 3/5

P(1^{st} and 2^{nd} green)= 9/25????

Nope!

They are dependent events

P(1^{st} and 2^{nd} green) = P(1^{st}) * P(2^{nd} green | 1^{st} green)

3/5 * 2/4 = 3/10

One final example

There are 300 students in the CS department. Of these students 90 play soccer, 30 play basketball, and 10 play both soccer and basketball. Let A be the event that a randomly selected student plays soccer and B be the event that the student plays basketball.

What is $P(A)$?

What is $P(B)$?

What is $P(A \text{ and } B)$?

What is $P(A|B)$?

Project 2

CORGIS dataset: The **C**ollection of **R**eally **G**reat, **I**nteresting, **S**ituated Datasets

https://think.cs.vt.edu/corgis/python/index.html